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Ablative melting of a solid cylinder perpendicularly pressed against a heated wall

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Abstract—This paper reports an analytical and experimental study on ablative melting of a solid cylinder perpendicularly pressed against a stationary heated surface. An explicit analytic solution is found for the rate of ablation in terms of temperature difference and pressure applied and of geometrical as well as physical properties of the solid and liquid. Data obtained in a limited number of rather crude experiments with rods of melting solids (ice, paraffin) and with rods of wood under flash pyrolysis conditions show a fair agreement with the predictions of the theoretical study thus confirming the "fusion model" of flash pyrolysis of wood in ablation regime.

INTRODUCTION

WITHIN the frame of a research program on the possibilities of upgrading the energy contained in biomass by thermal processes [1] the flash pyrolysis of wood has been studied by direct observation of the rate of ablation of a wood rod in contact with a hot surface [2]. The experiments have been carried out with cylindrical rods of beechwood (diameters from 2–10 mm) applied vertically to the upper horizontal surface of a spinning steel disc under known and variable pressures (0.1–3.5 MPa). The disc was heated from below by four gas burners to maintain surface temperatures on the disc constant (in the range from 773 to 1173 K). To prevent inflammation of the liquid and gaseous products a jet of argon was directed towards the contact surface. Under these conditions the rate of ablation, i.e. the velocity of consumption of the wood rods was found to be directly proportional to the wall temperature and to the pressure applied. For sufficiently high rotational velocities of the disc (> 1.5 m s^{-1} at the position of the rod, about 25 mm from the axis of the disc) neither the disc velocity nor the diameter of the rods had any significant influence on the rate of ablation [2].

Later these experiments on the spinning disc have been repeated with melting solids, such as ice, paraffin, lead and Rilsan [3]. The results were similar in all these cases, only the influence of pressure applied on the rods was found to be lower than for wood. The ablation velocity (v) was found to be proportional to the pressure (p) to a power of F with an exponent F depending on the substance ($F \approx 0.04$ for ice, 0.3 for paraffin, 0.7 for lead, 0.8 for Rilsan and ≈ 1.0 for wood)

$$v \sim p^F.$$

The reasons for this dependency have not yet been fully understood.

From these experiments, it was concluded that the rate was controlled by heat transfer through a thin layer of liquid products. Flash pyrolysis of wood could then be treated, at least to a first approximation as a simple fusion process with a "temperature of fusion" of wood of 739 K.

In case of a stationary heated wall the exponent F was found to be ~ 0.25 for all melting substances and also for the flash pyrolysis of wood.

In the following we present a theoretical treatment for the case of ablative melting of a cylindrical rod on a fixed heated wall. The results from this theory will be compared with data obtained in a number of experiments with rods of ice and paraffin as well as wood on a stationary heated surface.

THEORY

The starting point of our theoretical treatment is the situation schematically shown in Fig. 1. A cylindrical solid with a diameter $2R$ and a length much greater than $2R$ is continuously pressed against a horizontal wall maintained at a temperature T_W above the temperature of fusion T_f of the solid. In steady state a liquid layer of thickness s will be formed with a radial flow of liquid. For creeping flow, i.e. low Reynolds numbers the inertial forces can be neglected and it can

NOMENCLATURE

- C_p specific heat capacity at constant pressure ($J\,kg^{-1}\,K^{-1}$)
- F exponent
- g gravitational acceleration ($m\,s^{-2}$)
- h heat transfer coefficient ($W\,m^{-2}\,K^{-1}$)
- H specific enthalpy ($J\,kg^{-1}$)
- L height of a vertical wall (m)
- p pressure applied on the cylinder (Pa)
- p_0 atmospheric pressure (surroundings) (Pa)
- p^* characteristic reference pressure [$p^* = (\mu_l \alpha_l / R^2)$]
- P reduced pressure [$P = (p/p^*)$]
- Pe Peclet number
- Ph phase change number
- P_r local pressure at r (Pa)
- q heat flux ($W\,m^{-2}$)
- r radial coordinate (m)
- R radius of the solid cylinder (R)
- s thickness of liquid layer (m)
- T temperature (K)
- v ablation velocity of the solid cylinder ($m\,s^{-1}$)
- v_0 axial liquid velocity at $z = s$ ($T = T_f$) ($m\,s^{-1}$)
- v_r radial component of liquid velocity ($m\,s^{-1}$)
- v_z axial component of liquid velocity ($m\,s^{-1}$)
- v^* characteristic velocity [$v^* = (\alpha_l/R)$] ($m\,s^{-1}$)
- V reduced velocity [$V = (\rho_s/\rho_l)(v/v^*)$]
- y variable of integration
- z axial coordinate (m).

Greek symbols

- α thermal diffusivity ($m^2\,s^{-1}$)
- ϕ function defined by equation (13)
- λ thermal conductivity ($W\,m^{-1}\,K^{-1}$)
- μ viscosity of liquid (Pa s)
- ζ dimensionless axial coordinate [$\zeta = (z/s)$]
- ρ density ($kg\,m^{-3}$)

Subscripts

- f fusion
- l liquid (or fluid)
- r radial
- s solid
- W wall
- z axial

be shown† that the Navier–Stokes equations have the asymptotic solutions ($Re \to 0$):

$$v_r = 3v_0 \frac{r}{s} \zeta(1-\zeta) \tag{1}$$

$$v_z = -3v_0 \zeta^2 \left(1 - \frac{2}{3}\zeta\right), \tag{2}$$

where $\zeta = z/s$, v_r and v_z denote the radial and the axial components respectively of the velocity vector, v_0 is the axial velocity of the liquid at the position $z = s$ or $\zeta = 1$.

Owing to continuity of the mass flux at the melting interface, $\rho_s v = \rho_l v_0$, where v is the observed rate of ablation.

From these equations one can calculate the radial distribution of pressure:

$$p_r = p_0 + 3\mu_l v_0 \frac{R^2}{s^3}[1-(r/R)^2] \tag{3}$$

and the force acting on the solid cylinders in axial direction ($\pi R^2 p$), where p (without subscript) denotes the average pressure to be applied on the cylinder:

$$p = \frac{3}{2}\mu_l v_0 \frac{R^2}{s^3}. \tag{4}$$

From this hydrodynamic treatment one obtains a first relationship between the ablation velocity

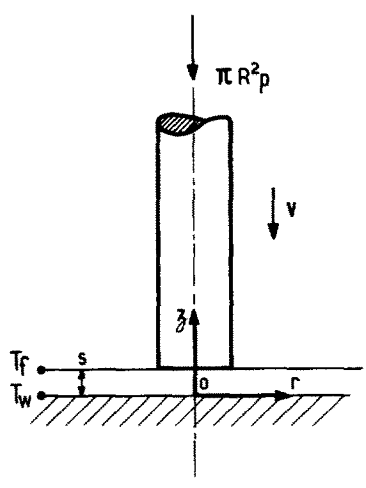

FIG. 1. Ablative melting of a solid cylinder pressed against a heated surface.

† The details of the calculations are to be found in the Appendix A.

$v = (\rho_l/\rho_s)v_0$, the pressure applied on the cylinder p, the cylinder radius R, the liquid viscosity μ_l and the thickness s of the liquid layer:

$$v_0 = \frac{2}{3} \cdot \frac{ps^3}{R^2\mu_l}. \tag{5}$$

The thickness s, however, in turn depends on v_0 as can be seen from the energy balance around the solid cylinder:

$$q_{(z=s)} = v_0\rho_l[H_l(T_f) - H_s(T_\infty)] \tag{6}$$

in combination with a first approximation for the heat flux

$$q_{(z=s)} \simeq \frac{\lambda_l}{s}(T_W - T_f), \tag{7}$$

leading to

$$s \simeq \frac{\alpha_l}{v_0} \frac{C_{pl}(T_W - T_f)}{\Delta H} \tag{8}$$

or

$$Pe \equiv \frac{v_0 \cdot s}{\alpha_l} \simeq \frac{C_{pl}(T_W - T_f)}{\Delta H} \equiv Ph. \tag{8a}$$

Elimination of s in equation (5) by (8) leads to

$$v_0 = \left(\frac{2}{3} \frac{\alpha_l^3}{\mu_l R^2} Ph^3 p\right)^{1/4}. \tag{9}$$

Equation (9) states, that the velocity of ablative melting of a cylinder is proportional to the pressure to a power of 1/4, to the radius to a power of $-1/2$ and to the temperature difference to a power of 3/4 in this first approximation.

It should be mentioned that this is analogous to the well known Nusselt formula for film condensation on a vertical surface [4], a fact that may be seen more easily, if we introduce a heat transfer coefficient h:

$$h = \rho_l v_0 \Delta H/(T_W - T_f) \tag{10}$$

$$h \simeq \left(\frac{2}{3} \frac{\lambda_l^3 \rho_l p \Delta H}{\mu_l R^2 (T_W - T_f)}\right)^{1/4}. \tag{11}$$

In Nusselt's formula p is replaced by $g\rho_l L$, R by L, and the numerical constant (2/3) is slightly different due to geometrical reasons.

In the case at hand it is not necessary, however, to use the somewhat crude approximation equation (7) for the heat flux at $z = s$, which neglects the fact, that there is a convective flow of energy in axial direction.

With equation (2) for the axial velocity, the energy equation can be solved (see Appendix B) to give

$$\frac{T_W - T(\zeta)}{T_W - T_f} = \frac{\phi(\zeta, Pe)}{\phi(1, Pe)}, \tag{12}$$

with

$$\phi(\zeta, Pe) = \int_0^\zeta \exp\left(-\frac{Pe}{2}(2y^3 - y^4)\right) dy \tag{13}$$

for the temperature T as a function of the dimensionless axial distance ζ in the liquid layer.

From the energy balance equation (6) and Fouriers law

$$q_{z=s} = -\lambda_l \left(\frac{dT}{dz}\right)_{z=s} \tag{14}$$

one obtains in place of equation (8a):

$$Pe = \frac{\exp(-Pe/2)}{\phi(1, Pe)} \cdot Ph \tag{15}$$

or solved for Ph and slightly rearranged:

$$Ph = Pe \int_0^1 \exp[Pe \cdot f(y)] dy \tag{15a}$$

$$f(y) = [1 - y^3(2 - y)]/2. \tag{15b}$$

Figure 2 shows a graph of equation (15) (full line) together with the Nusselt type approximation equa-

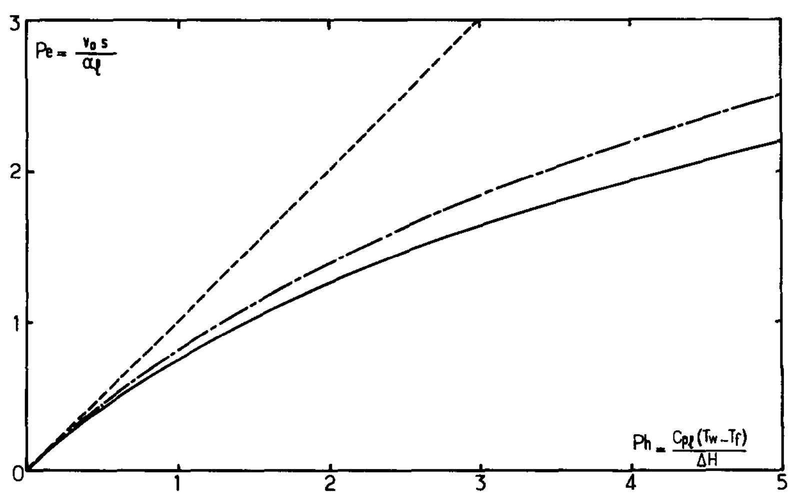

FIG. 2. Peclet number Pe vs phase change number Ph. ——— Nusselt type approximation. —·—·— Approximation by integral method. ——— Exact analytic solution.

tion (8a) (broken line) and an intermediate approximation obtained earlier with an integral method, using a cubic parabola [in place of the rigorous solution (12)] for the temperature profile.

It may be noticed, that the Nusselt type approximation should not be used except for very low values of the phase change number Ph.

In place of the approximate equation (9) we can now give the full solution:

$$v = \frac{\rho_l}{\rho_s}\left[\frac{2}{3}\frac{\alpha_l^3}{\mu_l R^2} Pe^3(Ph)p\right]^{1/4}. \tag{16}$$

The only difference to equation (9) being that the phase change number (or the temperature difference) is replaced by the function $Pe(Ph)$ as given by equation (15) (see Fig. 2).

It is possible to write equation (16) in dimensionless form. From dimensional analysis, two reference quantities may be defined: a characteristic velocity v^*

$$v^* = \alpha_l/R \tag{17}$$

and a characteristic pressure p^*

$$p^* = \mu_l v^*/R = \mu_l \alpha_l/R^2, \tag{18}$$

where v^* is the rate of heat diffusion across a distance R, p^* is the viscous drag per unit surface corresponding to a velocity gradient v^*/R. Defining reduced variables

$$V = \frac{\rho_s v}{\rho_l v^*} = \frac{\rho_s R}{\rho_l \alpha_l}\cdot v \tag{19}$$

and

$$P = \frac{p}{p^*} = \frac{R^2}{\mu_l \alpha_l}\cdot p \tag{20}$$

equation (16) may be rewritten as a "universal" relationship

$$V = \left[\frac{2}{3}Pe^3(Ph)P\right]^{1/4} \tag{21}$$

where $Ph = (T_W - T_f)Cp_l/\Delta H$ and the function $Pe(Ph)$ is given by equation (15). To avoid the somewhat tedious numerical integration [equations (15a, b)], the function $Pe(Ph)$ can be found from the simple approximation

$$Pe(Ph) \simeq \frac{Ph}{1+Ph^{5/6}/3}, \tag{22}$$

which fits the exact solution very well in the range $0 < Ph < 5$ as may be seen by comparison with Fig. 2. By plotting V against $[\frac{2}{3}Pe^3P]^{1/4}$ one should obtain a straight line of slope one.

The theoretical relationship for the rate of ablation as given by equations (16) or (21) together with (15) or (20) will be checked against some experimental data in the following section.

EXPERIMENTS

A few experiments have been carried out using a very simple set up as shown schematically in Fig. 3. A hexagonal hollow cylindrical piece of brass has been used to serve as the heated wall. Using a tubular nozzle, a small acetylene–oxygen flame was directed towards the inner surface directly adjacent to the upper horizontal plane of the hexagonal part. Parallel to this upper surface a small hole has been drilled into the piece, so that a chromel–alumel thermocouple (0.5 mm dia.) could be placed close to the surface. The cylinders made from ice, paraffin and beechwood were kept in a vertical position by a guiding tube. The pressure was varied by fixing different weights to the

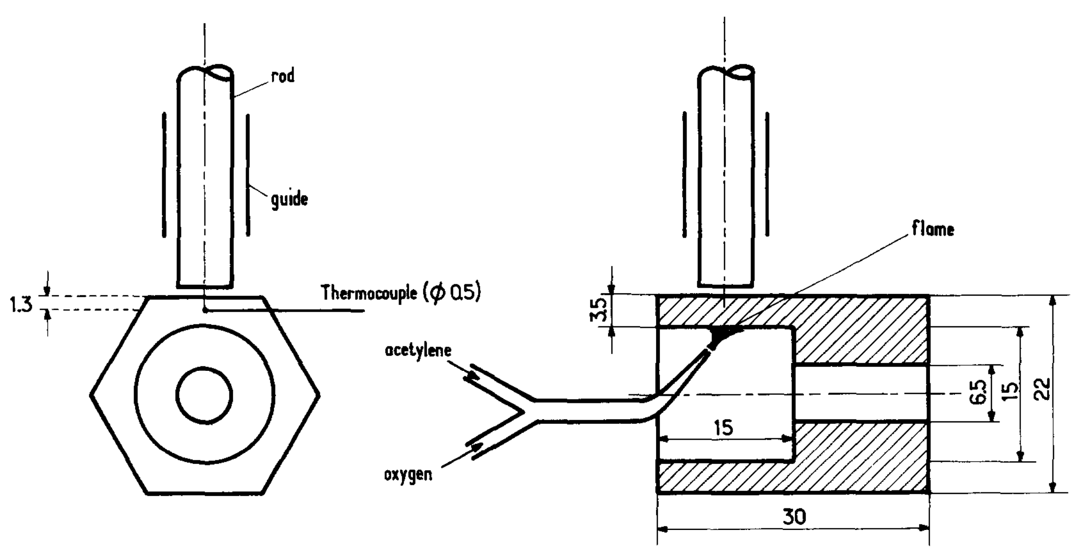

FIG. 3. The experimental set-up (all dimensions in mm).

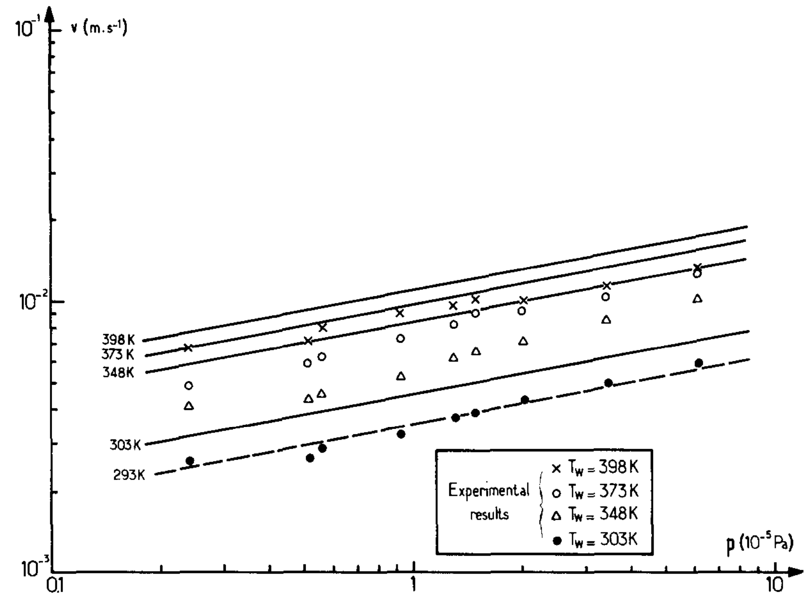

FIG. 4. Ablative melting velocity v vs pressure applied on the cylinder p with the wall temperature T_W as parameter: a comparison between theoretical straight lines and experimental points. Ice cylinders.

upper ends of the cylinders. The velocity of ablation was measured using two marks on the cylinder at a certain known distance with the fixed guiding tube as a frame of reference and a stopwatch to determine the time.

The data obtained are plotted on Figs. 4–6 as ablation velocity v vs pressure p with the measured wall temperature T_W (about 1.3 mm below the upper surface) as a parameter. The lines represent the predictions from equations (15) and (16) using the physical properties at T_f† as given in Table 1. Since the physical properties of the mixture of liquid, gaseous and solid products forming the fluid layer in the case of wood under flash pyrolysis conditions are not known, the comparison for wood in Fig. 6 is of a more qualitative nature. The factor containing the physical properties in equation (16) has been fitted to the experimental results. Nevertheless the variation of v with both the pressure p and the temperature difference $(T_W - T_f)$, taking a "temperature of fusion" of wood

† Due to the axial convection the average temperature must be in the range $T_f < T_{av} < \frac{1}{2}(T_f + T_W)$, therefore we used the physical properties at T_f for the sake of simplicity.

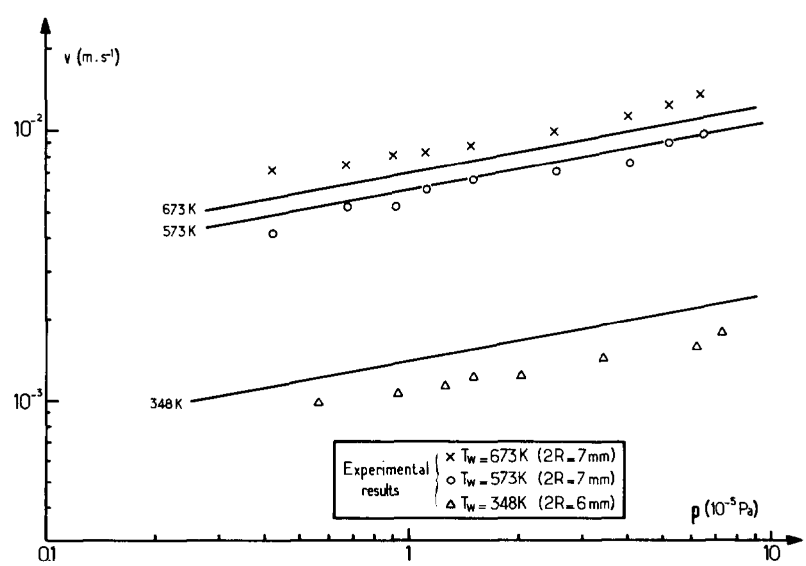

FIG. 5. Ablative melting velocity v vs pressure applied on the cylinder p with the wall temperature T_W as parameter: a comparison between theoretical straight lines and experimental points. Paraffin cylinders.

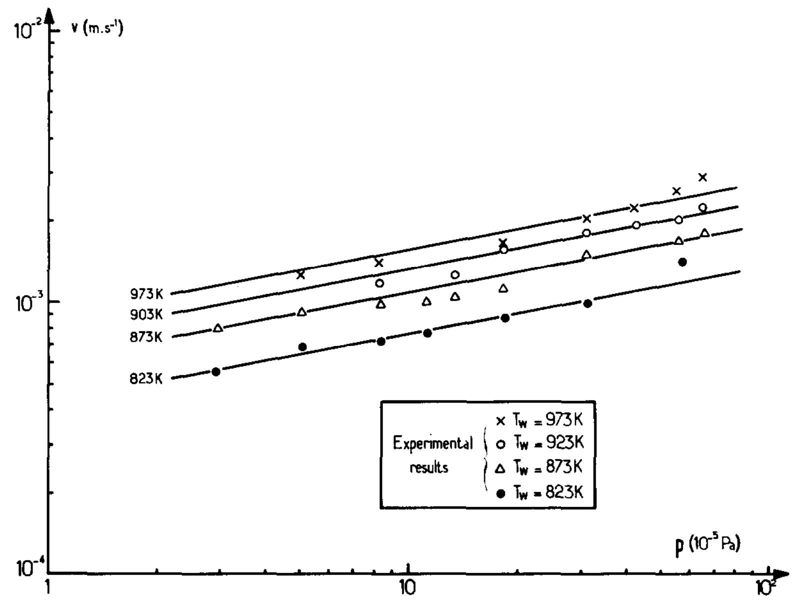

FIG. 6. Ablation velocity v vs pressure p with wall temperature T_W as parameter. Flash pyrolysis of beechwood.

[2] of $T_f = 739$ K, is very well represented by equations (15) and (16) using one single constant for the term containing the physical properties:

$$v_{(\text{wood}, R=10^{-3}\text{m})} = 1.55\, 10^{-3}\left(Pe^3 \frac{p}{p_0}\right)^{1/4} \text{m s}^{-1} \qquad (16\text{a})$$

with $P_0 = 10^5$ Pa and $Pe(Ph)$ from equation (15). The fitted constant 1.55×10^{-3} associated with estimated values for $\rho_l Cp_l$ and α_l (Table 1) allows to calculate $\mu_l = 72.5 \times 10^{-3}$ Pa s which seems a quite reasonable value for the viscosity of a liquid at a melting point of 739 K.

In the case of ice, where the physical properties are well known, the measured velocities of ablative melting are about 25% lower than those predicted from equations (15) and (16). This may be better seen from Fig. 7, where the data from Fig. 4 have been plotted as $v/(p/p_0)^{1/4}$ vs Ph or the temperature difference. Due to the relatively high heat fluxes the temperatures at the surface must have been considerably lower than those measured about 1.3 mm below the surface. The fact that there is a constant relative deviation between the measured and the calculated values confirms that an additional thermal resistance might well be the reason for it.

In order to check, whether the influence of the geometrical parameter R (the radius of the cylinder) is correctly predicted by equation (16), two series of runs with cylinders of paraffin with $2R = 6$ mm and $2R = 4$ mm have been carried out at the same wall temperature $T_W = 348$ K. The results are given in Table 2.

It may be seen that the ratio of the average ablation velocities for the 4 mm cylinders to the 6 mm cylinders comes very close to the value predicted from equation (16) ($\sqrt{6/4} \simeq 1.22$).

Finally all the data have been replotted according to the "universal" dimensionless relationship (21) in Fig. 8.

Table 1. Physical properties used for the calculations

	T_f	T_∞	ρ_s	ρ_l	C_{ps}	C_{pl}	ΔH_f	$(\Delta H/C_{pl})$†	μ_l	α_l
	(K)		(kg m^{-3})		(kJ kg^{-1} K^{-1})		(kJ kg^{-1})	(K)	(Pa s)	(m^2 s^{-1})
Ice (H_2O)	273	268	917	1000	2.0	4.2	333.5	81.78	1.792×10^{-3}	1.333×10^{-7}
Paraffin	328	293	890	780	1.9	2.7	147	79.07	4.3×10^{-3}	0.712×10^{-7}
Beechwood	"739"	293	720	500^c	2.8	3.65‖	40	411.5‡	72.5×10^{-3}§	0.3×10^{-7}‖

† $\Delta H = \Delta H_f(T_f) + C_{ps}(T_f - T_\infty)$.

‡ The moisture content of beechwood was determined to be $X = 0.0953$ (based on dry mass), therefore ΔH was calculated as:

$$\Delta H \simeq [40 + 2.8 \times 446 + 0.0953\ (2500 + 2.0 \times 446] = [1288.8 + 212.5] = 1501.3 \text{ kJ kg}^{-1}.$$

§ Fitted value.

‖ Estimated value [3] (the choice of these values is not very sensitive on the results for wood).

Table 2. Influence of diameter ($2R$) on velocity (v). Data for paraffin $T_f = 328$ K and $T = 293$ K

$T_W = 348$ K	p (10^{-5} Pa)	0.56	0.93	1.25	1.49	2.03	3.47	6.22	7.19	
$2R = 6 \times 10^{-3}$ m	$v \times 10^3$ (m s^{-1})	0.95	1.05	1.10	1.20	1.20	1.43	1.56	1.76	
	$v/p^{1/4} \times 10^3$	1.098	1.069	1.040	1.086	1.005	1.048	0.988	1.075	
	$(v/p^{1/4})$ average	1.05×10^{-3} (+4.6%, −5.9%)								
$T_W = 348$ K	p (10^{-5} Pa)	0.73	1.30	2.09	2.81	3.36	4.57	7.81	14.0	16.2
$2R = 4 \times 10^{-3}$ m	$v \times 10^3$ (m s^{-1})	1.25	1.33	1.46	1.54	1.67	1.89	2.11	2.45	2.76
	$v/p^{1/4} \times 10^3$	1.352	1.246	1.214	1.189	1.233	1.293	1.262	1.267	1.376
	$(v/p^{1/4})$ average	1.27×10^{-3} (+8.3%, −6.4%)								

$\bar{v}_{4\times10^{-3}\text{m}}/\bar{v}_{6\times10^{-3}\text{m}} = 1.21 \quad (\bar{v}_{4\times10^{-3}\text{m}}/\bar{v}_{6\times10^{-3}\text{m}})_{\text{theory}} = 1.22$

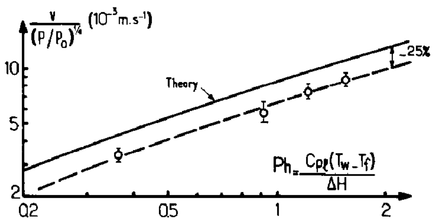

FIG. 7. Ablative melting velocity as a function of temperature difference (or phase change number) for ice cylinder ($p_0 = 10^5$ Pa).

As already shown in the previous figures the agreement between theory and the results of all the experiments is fairly good. The possible reasons for the somewhat larger deviations for the data with ice have been discussed already in connection with Figs. 4 and 7.

CONCLUSIONS

A creeping flow solution of the Navier–Stokes and the energy equations was obtained for ablative melting of a cylinder perpendicularly pressed against a heated surface with a constant force. From this explicit analytic solution, which is analogous, in the limit of small temperature differences, to the well known Nusselt-formula for filmwise condensation of vapour on a vertical surface, the ablation velocity is found to be proportional to the fourth root of the pressure applied and inversely proportional to the square root of the diameter of the cylinder. For small temperature differences—wall temperature minus temperature of fusion—the ablation velocity is proportional to the temperature difference to a power of 3/4. The results of some simple experiments with ice and paraffin as solids confirmed the theoretical predictions. Experiments with cylinders of beechwood under flash pyrolysis conditions gave the same relationships between ablation velocity, temperature and pressure. At last, a universal relationship is proposed to correlate dimensionless values of ablation rate and applied pressure.

Note added in proof

Recently Saito *et al.* have published two reports about an experimental [5] and a numerical [6] study "on the contact

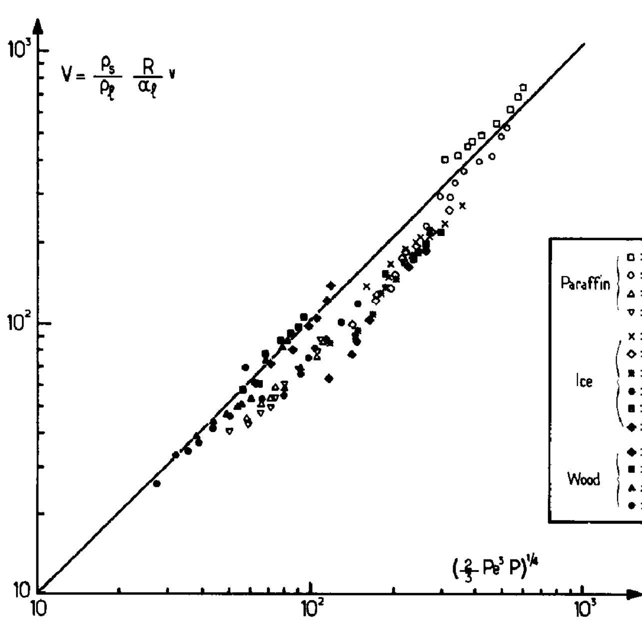

FIG. 8. "Universal" dimensionless correlation of the whole set of experimental results.

heat transfer with melting". They used cylinders of ice and of octadecane (with 50 and 100 mm diameter) pressed against an electrically heated plate [5]. In their second report [6] they solved the momentum and energy equations (neglecting the inertia forces as we did in our analysis) by a numerical (finite-difference) method. They presented their results as a nondimensional heat flux $q^* = q_{(z=0)}R/(\lambda\Delta T)$ in terms of a dimensionless pressure $pR^2/\mu\alpha$ and a Stefan number Ste. The heat flux at the heated plate exceeds the flux at the phase change boundary by a factor $e^{Pe/2}$ [as may be seen from our equations (12)–(15)].

With the dimensionless pressure corresponding exactly to our $P = pR^2/\mu\alpha$ and the Stefan number being equal to our Ph one can easily obtain q^* from our equations (21), (22) for $V(Pe, P)$ and $Pe(Ph)$ as:

$$q^* = e^{Pe/2}\, V/Ph. \tag{23}$$

For low values of Ph (or $Ste < 0.1$) the authors obtained a formula from a least square fit of their numerical results which can be written (in our notation) as

$$q^* = 0.94\, P^{0.25}\, Ph^{-0.25} \quad (Ph < 0.1).$$

From our analytical solution one obtains (with $Pe \to Ph$ for small values of Ph) an asymptotic relationship:

$$q^* = \left(\frac{2}{3}\right)^{1/4} P^{1/4}\, Ph^{-1/4} \quad (Ph \ll 1).$$

With $(2/3)^{1/4} = 0.9036$, i.e. 4% lower than the numerical constant found for $10^{-3} < Ph < 10^{-1}$ by Saito *et al.*, these are in excellent agreement with each other. Values calculated from the more general equation (23) with (21), (22) were found to agree almost perfectly in the whole range of $10^{-3} < Ph < 2$ presented in graphical form in Fig. 10 of ref. [6]. As the experiments from ref. [5] show good agreement with the numerical results of ref. [6] they do so with our analytic solution too. It is not quite clear, however, why the authors in their numerical study found a slight deformation of the phase change boundary, with a larger thickness of the liquid layer at the center, for the highest Stefan numbers investigated (see Fig. 9 in [6]). As far as we know there is no experimental evidence for such a deformation and our analytic solution with the temperature being a function of z only seems to be in good agreement with both experimental and numerical results.

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APPENDIX A

Hydrodynamic solution

For steady state and constant physical properties the conservation equations for mass and momentum may be written as:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{\partial}{\partial z}v_z = 0 \tag{A1}$$

$$-\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{\partial^2 v_r}{\partial z^2}\right] = 0 \tag{A2}$$

$$(Re \to 0)$$

$$-\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{\partial^2 v_z}{\partial z^2}\right] = 0. \tag{A3}$$

With the boundary conditions:

$$v_z(z = s) = -v_0 = -(\rho_s/\rho_l)v \tag{A4}$$

$$v_r(z = s) = 0 \tag{A5}$$

$$v_z(z = 0) = 0 \tag{A6}$$

$$v_r(z = 0) = 0. \tag{A7}$$

The mass balance around the liquid cylinder of height s and radius r gives:

$$\bar{v}_r = r \cdot v_0/(2 \cdot s). \tag{A8}$$

We are therefore trying to find a solution of the form

$$v_r = r \cdot f(z) \tag{A9}$$

$$v_z = g(z). \tag{A10}$$

From (A9) and (A1) one obtains:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) = -\frac{\partial v_z}{\partial z} = 2f(z) = -g'(z), \tag{A11}$$

which makes the first terms in brackets in equations (A2) and (A3) equal to zero:

$$-\frac{\partial p}{\partial r} + \mu r f''(z) = 0 \tag{A12}$$

$$-\frac{\partial p}{\partial z} - \mu 2 f'(z) = 0. \tag{A13}$$

With

$$\frac{\partial^2 p}{\partial r \partial z} = \frac{\partial^2 p}{\partial z \partial r}$$

the function $f(z)$ has to fulfil the condition

$$f'''(z) = 0$$

leading to a parabolic velocity profile:

$$f(z) = 3(v_0/s)\frac{z}{s}\left(1 - \frac{z}{s}\right). \tag{A14}$$

(The conditions A5, A7 and A8 have been used to determine the constants.) From this the final solutions as given in equations (1)–(4) in the text are easily obtained.

APPENDIX B

Thermal solution

The conservation of energy may be expressed by

$$v_z \frac{dT}{dz} = \alpha \frac{d^2T}{dz^2}, \qquad \text{(B1)}$$

(T is a function of z only) with v_z from equation (2), equation (B1) can be solved by separation of variables:

$$\frac{d^2T/d\zeta^2}{dt/d\zeta} = -Pe(3\zeta^2 - 2\zeta^3) \qquad \text{(B2)}$$

$$\ln \frac{dT}{d\zeta} = -Pe\left(\zeta^3 - \frac{1}{2}\zeta^4\right) + \ln C_1 \qquad \text{(B3)}$$

$$T = C_1 \int_0^1 \exp\left[-Pe\left(\zeta^3 - \frac{1}{2}\zeta^4\right)\right] d\zeta + C_2. \qquad \text{(B4)}$$

With the boundary conditions:

$$T(\zeta = 0) = T_w \qquad \text{(B5)}$$

$$T(\zeta = 1) = T_f, \qquad \text{(B6)}$$

then the solution given as equation (12) in the text is obtained.

FUSION EN REGIME D'ABLATION D'UN CYLINDRE SOLIDE APPLIQUE PERPENDICULAIREMENT SUR UNE PAROI CHAUDE

Résumé—Cet article présente une étude théorique et expérimentale sur la fusion en régime d'ablation d'un cylindre appliqué sous une certaine pression sur une surface fixe chauffée. On propose une solution analytique pour le calcul de la vitesse d'ablation en fonction de la différence de température (température de la surface, température de fusion), de la pression appliquée, de la géométrie et des propriétés physiques du solide et du liquide. Les résultats d'expériences effectuées avec des baguettes de solide en fusion (glace, paraffine) et avec des baguettes de bois dans des conditions de pyrolyse éclair montrent un bon accord avec les prévisions de l'étude théorique apportant une confirmation de la théorie du "modèle de fusion" relatif à la pyrolyse éclair du bois en régime d'ablation.

ABSCHMELZEN EINES FESTEN ZYLINDERS, DER SENKRECHT GEGEN EINE BEHEIZTE WAND GEPRESST WIRD

Zusammenfassung—Es wird über eine analytische und experimentelle Untersuchung berichtet zum Abschmelzen eines festen Zylinders, der gegen eine ruhende beheizte Wand gepresst wird. Eine geschlossene analytische Lösung für die Abschwelzgeschwindigkeit wurde gefunden in Abhängigheit der Temperaturdifferenz und des Anpressdruches sowie der geometrischen und stofflichen Eigenschaften des Feststoffes und der Schmelze. Daten aus einer begrenzten Zahl relativ einfacher Versuche mit Stäben aus schmelzenden Feststoffen (Eis, Paraffin) und mit Holzstäben im Bereich der schnellen ("Flash") Pyrolyse (unter Juertgas) zeigen eine brauchbare übereinstimmung mit den Voraussagen der Theorie und bestätigen damit das "Schmelz-Modell" der schnellen Pyrolyse von Holz.

АБЛЯЦИОННОЕ ПЛАВЛЕНИЕ ТВЕРДОТЕЛЬНОГО ЦИЛИНДРА, ПРИЖАТОГО К НАГРЕТОЙ СТЕНКЕ ПОД ПРЯМЫМ УГЛОМ

Аннотация—Аналитически и экспериментально исследовано абляционное плавление твердотельного цилиндра, прижатого к стационарно нагреваемой поверхности под прямым углом. Получено аналитическое решение в явном виде для скорости разрушения, определяемой через разность температур и прилагаемое давление, а также геометрические и физические свойства твердого тела и жидкости. Результаты, полученные из ограниченного числа довольно грубых экспериментов, проведенных со стержнями из плавящихся твердых тел (лед, парафин) и деревянными стержнями при условиях пиролиза, показывают хорошее соответствие с расчетными данными, подтверждая таким образом "модель плавления" пиролиза дерева в режиме абляции.